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ABSTRACT

Criterion-referenced testing is an important area in the theory and practice of educational measurement. This study demonstrated that even these tests must be closely examined for construct validity. The dimensionality of a dataset will be affected by the examinee's cognitive processes as well as by the nature of the content domain. The methods of extracting a unidimensional subset from an achievement dataset were studied. A second purpose was to apply a general technique for detecting aberrant response patterns derived from wrong rules of operation. The Individual Consistency Index (ICI) was found effective in detecting the anomalous response patterns resulting from some misconceptions. However, it requires repeated measures. Applicability to tests that do not have several parallel items to measure the performance of a single task will be limited. Although computerized error diagnostic programs can identify misconceptions possessed by a student in the very specific domain of arithmetic, ICI can be applicable to more general domains. It can detect candidates to route to the expensive error-diagnostic programs. (Author/DWH)

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SPOTTING INCORRECT RULES IN SIGNED-NUMBER ARITHMETIC BY THE INDIVIDUAL CONSISTENCY INDEX

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Abstract

This study demonstrates that even a criterion-referenced test, in which items are chosen from a single content domain, requires a close examination of construct validity. The Individual Consistency Index (ICI) is effective in detecting anomalous response patterns resulting from some misconception(s). The subset obtained by deleting the responses which were spotted by ICI showed a higher unidimensionality. The same result was replicated by another dataset whose test was parallel but not identical to the previous one. Although computerized error diagnostic programs can identify misconceptions possessed by a student in the very specific domain of arithmetic, ICI can be applicable to more general domains and detect possible candidates to route to the expensive error-diagnostic programs.

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Replace reference of Tatsuoka, K. K., & Tatsuoka M. M. (Research Report 81-4) with

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Spotting Erroneous Rules of Operation

by the Individual Consistency Index

Introduction

Item Response Curve Theory (IRT) has proved its important role in modern testing practices such as computerized adaptive testing, in which each examinee takes a different set of items. The student's ability level is estimated and located as a point on a continuum. However, a drawback of IRT models that are presently available in practice is the constraint of unidimensionality on datasets (Lord & Novick, 1968). Reckase (1979) warned of and demonstrated the serious consequences for parameter estimation resulting from the violation of unidimensionality. It has been observed that the scores obtained from most achievement tests, unlike ability tests, are affected by two or more latent trait variables.

Moreover, Tatsuoka and Birenbaum (1979, 1981) showed that the dimensionaltiy of a dataset obtained from the middle learning stages, when the students are still far from mastery, was multidimensional even though the test items are clearly drawn from a single domain. Their result indicates that a close examination of the construct validity of criterion-referenced tests is necessary.

Brown & Burton (1978) developed an error-diagnostic model for whole-number subtraction problems. Their model "BUGGY" showed that wrong rules can yield the correct answers in some test items. Birenbaum & Tatsuoka (1980) found that 1 - 0 scoring based simply on right or wrong answers caused serious problems when erroneous rules of signed-number operations were used by many examinees.

Using data from a 64-item test consisting of four parallel subtests of 16 items each, Birenbaum & Tatsuoka first did a principal components analysis on the original data -- with the items scored 1 or 0 in the usual manner. Next, the data were modified by giving a score of 0 when an item was correctly answered presumably by use of an erroneous rule, and another principal component analysis was done. The change between the two analyses was dramatic. The dimensionality of the data became much more clearcut with the modified data. The item-total correlations became much higher, while the means of the 16 tasks (each represented by four parallel items) did not change significantly. The above phenomenon suggests why some achievement tests cannot by treated as unidimensional even though the items are taken from a single content domain.

One of the purposes of this paper is to investigate methods of extracting a unidimensional subset from an achievement dataset. Error analysis, which is usually performed by a series of clinical interviews and intuitive interpretation of the student's responses to the test items is a cumbersome work. At present, there are only a very few computer programs available for providing diagnoses of misconceptions possessed by students, such as Brown and Burton's "BUGGY", and "SIGNBUG" developed by Tatsuoka et al., (1980). But they are expensive, and moreover, they can handle only specific areas of arithmetic. Our intention is to find a more general technique applicable to other content areas in order to detect aberrant response patterns which are derived from erroneous rules of operation (or bugs).

The second purpose is to replicate the result described in the Birenbaum-Tatsuoka study by applying the general technique for detecting aberrant response patterns derived from wrong rules of operation.

It turned out that the index, Individual Consistency Index (ICI) introduced in Tatsuoka & Tatsuoka (1980) is very effective for spotting erroneous rules of operation in signed-number computation problems: The responses yielded by wrong rules are characterized by having low scores and high ICI values. Moreover, the subset obtained from the original dataset by deleting the subjects who have low total scores and high ICI values demonstrated exactly the same phenomenon, that is, the dimensionality of the subset became nearly unidimensional, as the modified data did. The structure of the subset in terms of cognitive performance is interesting. It consists of the responses produced by using the right rule and errors probably committed by students randomly or inconsistently. The result will be useful for understanding the meaning of dimensionality of achievement data. It also shows the importance of construct validity, even in criterion referenced testing of the cognitive aspect of performance, and that the traditional means of item analysis that are based on taking the variances of binary scores and content analysis into consideration are not enough for constructing test items that are capable of diagnosing misconceptions.

Method and Procedure

Is 1-0 scoring justifiable?

A test containing 64 signed-number addition and subtraction problems, consisting of four parallel subtests of 16 items each, was administered to 127 eighth graders at a local junior high school after the instruction was completed. (This test will be referred to as the "November data" hereafter.) Each item in the test was carefully related so as to maximize the capability of diagnosing erroneous rules of operation. In signed-number computation, 98% of students' responses are summarized by four types: adding or subtracting two absolute values and putting the ~~sign of plus~~ or minus on answers. Nine problem types in subtraction and six in addition (see Appendix I) are the necessary minimum number of items in order to maintain the error diagnostic capability for providing a specific description of a vast majority of popular errors.

Tatsuoka et al. (1980) developed an error-diagnostic system called "SIGNBUG" for signed number problems on the PLATO system at the University of Illinois. With this computer program, the performance of the 127 students was thoroughly analyzed.

The same test was administered to 180 seventh graders who were still far from the mastery stage and exhibited a variety of confusion in the material. (Data from this test will be referred to as the "January data" hereafter). The responses to the items in the January data were also analyzed by "SIGNBUG" and their complete erroneous rules of operation with those from the November data are described in Appendices I and II. Actually we have found many more erroneous rules, incomplete ones and those applicable only to addition problems, but the number of bugs in the list is limited to the erroneous rules that appeared in subtraction problems. The addition items form another dimension by both a principal components analysis, and a multidimensional scaling (Birenbaum & Tatsuoka, 1980) after the modification procedure described earlier was taken. Therefore we chose the subtraction problems for our study.

Insert Table 1 about here

Table 1 is the list of the binary scores on 15 tasks (i.e., problem types) yielded by the 45 bugs given in Appendix II. The rightmost column shows the total scores on 15 tasks. The first and second numbers in the parentheses are the total scores on addition and subtraction tasks, respectively. The bottom line of Table 1 contains the total number of 1's for each item type. For example, for the task -16 - (-7), the correct answer is yielded by 26 out of 42 erroneous rules. If the data is collected while many students are confused by a variety of errors, then this task, No. 8, will have the highest number of 1's. Thus, although task No. 8 turned out to be the "easiest" in the January data, this must be partly attributed to the fact that the correct answer can be obtained by so many erroneous rules. Table 2 shows the rank orders in the proportion correct of 12 tasks for the January and the modified November data (as described earlier). However, the

Insert Table 2 about here

counterpart of this task, -6 - (-8) has only twelve 1's out of 42. These items should almost be equally difficult with respect to the conceptualization of the subtraction problems because of the teaching method. But their positions of the item difficulty order in the datasets (both November and January data) are quite different. The descending order of the total 1's over 15 tasks in Table 1 is 8, 15, 7, 16, 11, 5. These six items are in the top seven items having the largest number of the total 1's in the January data. As mentioned earlier, 80% of the examinees in the January data used a variety of erroneous rules of operation.

Importance of Item Ordering

Harnisch and Linn, (1981) classified indices that measure the degree to which an individual response pattern is atypical into two different types. One type consists of those formulated by using the orders of

Table 1

Observed Complete Rules and their Response Patterns
for Signed Number Addition and Subtraction Problems

Bugs	Addition						Subtraction									Total Scores
	3	5	10	11	14	15	1	2	4	7	8	9	12	13	16	
1 or 2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	15 (6,9)
3	1	1	1	1	1	1	0	1	0	1	0	1	0	1	1	11 (6,5)
4	1	1	1	1	1	1	1	0	1	0	1	0	1	1	0	11 (6,5)
5	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	6 (6,0)
6	1	1	1	1	1	1	0	0	0	1	0	0	0	0	1	8 (6,2)
7 or 8	1	1	1	1	1	1	0	1	0	1	0	1	0	0	1	10 (6,4)
9	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	6 (6,0)
10	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	6 (6,0)
11	1	1	1	1	1	1	1	0	1	0	1	0	1	1	0	11 (6,5)
12	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	13 (6,7)
13	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	6 (6,0)
14	1	1	1	1	1	1	1	0	1	0	1	0	1	0	0	10 (6,4)
15	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	14 (6,8)
16	1	1	1	1	1	1	0	0	0	1	1	0	0	0	0	8 (6,2)
17	1	1	1	1	1	1	1	0	0	1	1	0	0	0	0	9 (6,3)
18	1	1	1	1	1	1	0	0	0	1	1	0	0	0	0	8 (6,2)
19	0	0	1	0	1	0	0	1	1	0	0	1	1	1	0	7 (2,5)
20	0	1	0	1	1	1	1	0	0	1	1	0	0	0	1	8 (4,4)
21	1	1	1	1	1	1	0	1	0	0	0	1	0	1	0	9 (6,3)
22	1	1	1	1	1	1	0	0	0	0	1	0	0	0	1	8 (6,2)
23	0	1	0	1	0	1	0	0	0	1	1	0	0	0	0	5 (3,2)
24	0	1	0	1	0	1	0	0	0	0	1	0	0	0	1	5 (3,2)
25	0	1	0	1	0	1	0	0	0	1	0	0	0	0	1	5 (3,2)
26	1	0	0	1	0	1	0	0	0	0	1	0	0	0	1	5 (3,2)
27	0	0	1	0	0	0	0	0	0	0	1	0	0	0	1	3 (1,2)
28	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	2 (0,2)
29	0	1	0	0	0	0	0	0	0	0	1	0	0	0	1	3 (1,2)
30	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	3 (1,2)
31	0	0	0	0	0	1	0	0	0	0	1	0	0	0	1	3 (1,2)
32	0	1	0	0	0	0	1	0	0	1	0	0	0	0	0	3 (1,2)
33	0	0	1	0	1	0	0	0	0	1	1	0	0	0	0	4 (2,2)
34	0	0	0	0	0	1	0	0	1	0	1	0	1	0	0	4 (1,3)
35	0	0	0	0	0	1	0	0	0	0	1	0	0	0	1	3 (1,2)
36	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	2 (0,2)
37	1	0	0	1	0	0	0	0	0	0	1	0	0	0	1	4 (2,2)
38	0	0	1	1	1	0	0	1	0	1	0	1	0	0	0	6 (3,3)
39	0	0	1	0	1	0	0	0	0	1	1	0	0	0	0	4 (2,2)
40	1	1	1	1	1	1	1	0	0	1	1	0	0	0	0	9 (6,3)
41	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	9 (0,9)
42	0	1	0	1	0	1	0	1	1	0	0	1	1	1	0	8 (3,5)
43	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	3 (3,0)
44	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	6 (6,0)
45	1	1	1	1	1	1	0	1	0	0	0	1	0	0	0	8 (6,2)
Total 1's	24	28	26	30	26	32	12	10	10	20	26	10	10	9	18	
Proportion of 1's to # of bugs	3/*22	7/22	5/22	9/22	5/22	11/22	12/42	10/42	10/42	20/42	26/42	10/42	10/42	9/42	18/42	

*22 erroneous rules (or bugs) because 21 out of 43 bugs used the right rule for addition problems.

Table 2

Rank Orders of the Task Difficulties (from Easy to Hard) in Two Datasets[a]

Task Type	November[c] Modified	January Data[d]
12 + -3	3	7
-14 + -5	9	12
-3 + 12	5	6
-5+-7[b]	13	3
3 +-5	10	11
-6 + 4	14	5
8 - 6	6	9
-16 - (-7)	7	8
-6 - (-8)	1	1
-3 - +12	12	2
2 - 11	15	10
9 - (-7)[b]	11	4
1 - (-10)	4	
-7 - 9	2	
-12 - 3[b]	8	

[a]The task type 6 + 4 (item number 6 in Appendix I) has been omitted.

[b]These tasks were note included in the January test.

[c]The November order is based on the difficulties estimated from the IRT model.

[d]The January order is based on the actual proportion answering each item correct.

difficulty and the other comprises those based on the comparison of an individual response pattern to some kind of a standard response pattern. The former group consists of Van der Flier's index (1977), the Norm Conformity Index (Tatsuoka & Tatsuoka, 1980) and γ index (Sato, 1972). The latter contains Sato's Caution index (Sato, 1975), Linn & Harnisch's (1981) modified caution index and Kane & Brennan's (1978) coefficient of agreement. A weighted sum of NCI leads to Cliff's (Cliff, 1978) Consistency Index, C_{t2} (Tatsuoka & Tatsuoka, 1980) while the caution index has an algebraic relationship to Loevinger's homogeneity index (Takeya, 1978).

Although this classification is useful for pointing out a certain conceptual difference between the two types of indices, the fact remains that both types are dependent on the order in which the items are arranged in calculating them. This dependence on item order is made explicit in the case of the Norm Conformity Index (NCI), whose definition calls for arranging the items in descending order of difficulty for an <u>arbitrary</u> norm or reference group. The extent to which a given individual's response pattern then resembles a Guttman vector (in which all zeros precede all 1's) with the same number of 1's is what the NCI measures. The reference group may be the group of which the individual is a member, or it may be some other group of interest to the researcher.

On the other hand, the caution index was defined by Sato (1975) in the context of a data matrix in which the items are arranged in ascending order of difficulty for the group at hand and the individuals are arranged in descending order of total score. (Such a matrix is called an S-P table.) The question of dependence on item order does not normally arise since the caution index C_i is defined for this one particular arrangement of items only. It is the complement of the ratio of two covariances that will be specified later. C_i measures the extent to which the i-th individual's response pattern is atypical of the group of which he/she is a member. However, there is no reason why we cannot speak of the atypicality of an individual's response pattern compared to the average response pattern of some group other than the one to which he/she belongs. The order of items will then be different from before, and the value of C_i will change. Thus the caution index, too, is dependent on item order.

We wish to demonstrate the extent and way in which NCI and C_i are item-order dependent with reference to the set of 12 items shown in Table 2, that are common to both the November data and the January data, but have different difficulty orders in the two datasets. However, it may be useful first to give brief descriptions of the calculations for the two indices by using a smaller numerical example.

<u>Example</u>. Let us refer back to the 43 x 9 matrix of binary scores obtained by using the correct rule and 42 erroneous rules for solving nine subtraction problems, displayed in the right-hand panel of Table 1. We now pretend that this is a data matrix for a group of 43 examinees who took a nine-item test, and denote it as

$$Y = (y_{ij}), \quad i=1,2...,43; \; j=1,2,...,9 \quad .$$

Each element of the vector of column totals $[y_{.1}, y_{.2}, \ldots y_{.9}]$ is one greater than the corresponding column total shown in Table 1 (because the totals there excluded the 1's in the first row since that row represented the correct rule of operation, whereas Table 1 was concerned with 1's generated by erroneous rules). However, since we need to arrange the items in monotonic order of difficulty (descending order for NCI, ascending for C_i), let us renumber the items from 1 to 9 in the order they are to occur in the formulas.

For calculating NCI, then, the item order is (2, 9, 13, 12, 4, 1, 16, 7, 8). We take the second row (for Bug #3) as the response pattern of the examinee whose NCI we want to calculate. When the items are rearranged as just indicated, the response vector becomes

$$\underline{y}_3 = [1\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 0] \quad ,$$

whose elements we denote as $y_{31}, y_{32}, \ldots, y_{39}$ in the order they occur here.

We also need the total score $y_{3.}$, which is 5 in this case. In general,

$$y_{i.} = \sum_{j=1}^{n} y_{ij}$$

where n (= 9 here) is the number of items. The NCI for individual i may now be defined as

$$(NCI)_i = 2U_{ia}/U_i - 1 \quad ,$$

where

$$U_{ia} = \sum_{j=1}^{n-1} \sum_{k=j+1}^{n} (1-y_{ij})y_{ik}$$

and

$$U_i = y_{i.}(n-y_{i.})$$

[Verbally, U_{ia} means the sum of all 1's to the right of each 0 in the vector y_i, added over all 0's; this represents the number of (0,1) pairs that occur in $\underline{y}_i$. U_i, on the other hand, represents the total number of (0,1) and (1,0) pairs in $\underline{y}_i$.]

For our example,

$$U_{3a} = 2 + 2 + 2 = 6$$

(because there are three 0's, each with two 1's to its right; the last 0 does not have any number to its right and hence contributes nothing to the sum), while

$$U_3 = (5)(4) = 20 \quad .$$

Hence,

$$(NCI)_3 = (2)(6)/20 - 1 = -.4$$

(The negative sign indicates that the response pattern y_3 is closer to the reverse Guttman vector [1 1 1 1 1 0 0 0 0] than to the Guttman vector [0 0 0 0 1 1 1 1 1], which agrees with our impression on looking at y_3.)

We now illustrate the calculation of C_i for the same response pattern. Since the formula calls for the items to be arranged in ascending order of difficulty, we reverse the response vector to get

$$y_3 = [0\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 1]$$

We also need the vector of column totals

$$y_n = [y_{.1}, y_{.2}, \ldots, y_{.9}]$$

$$= [26, 20, 18, 12, 11, 10, 9, 9, 2] \quad .$$

Finally, the reverse Guttman vector with five 1's is

$$y_3^S = [1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0]$$

With the elements of these three vectors appropriately symbolized, and '3' replaced by i for generality, the formula for the caution index is

$$C_i = 1 - \frac{\text{cov}\ (y_{ij}, y_{.j})}{\text{cov}\ (y^S_{ij}, y_{.j})}$$

Making the substitutions, we get

$$\text{cov}(y_{3j}, y_{.j}) = [\sum_{j=1}^{9} y_{3j}y_{.j} - y_{3.} \sum_{j=1}^{9} y_{.j}/9]/8$$

$$= [(0)(26) + (1)(20) + (1)(18) + \ldots + (1)(2)$$

$$-(5)(26 + 20 + 18 + \ldots + 2)/9]/8$$

$$= -.875$$

and

$$\text{cov}(y_{3j}, y_{.j}) = [\sum_{j=1}^{9} y^S_{3j}y_{.j} - y^S_{3.} \sum_{j=1}^{9} y_{.j}/9]/8$$

$$= [(1)(26) + (1)(20) + (1)(18) + (1)(12) + (1)(11) + 0$$

$$-(5)(26 + 20 + 18 + \ldots + 2)/9]8$$

$$= 2.75$$

Hence, $C_3 = 1 - (-.875)/2.75 = 1.318$.

Item-Difficulty Orders in Early and Late Learning Stages.

We now examine the response patterns of three students taken from the January dataset, exhibit their NCI and C_1 values with items arranged in difficulty order for that dataset as well as in difficulty order for the November dataset, and note the extent and nature of the differences between the corresponding values. Although the two datasets are based on different samples, the results nevertheless give a general idea of the effect of using the difficulty orders in early and late learning stages. The relevant information is summarized in Tables 3 and 4.

Insert Table 3 about here

Student #37 is a very good student, who did most of the addition problems correctly, as seen in Table 4. About the only trouble she had was confusing parentheses with absolute-value bars, which is a relatively minor and easily remedied misconception. Yet, precisely because her misconception is a sophisticated one for students at this stage of learning, her response pattern is rare and atypical of the group. She thus gets a low NCI (-.10) and a high C_1 (.93), which are misleading because they imply that she needs to be cautioned and given remedial work.

Insert Table 4 about here

On the other hand, Students #12 and #30 possess misconceptions that are rather common in this dataset. Hence, their NCI values are relatively high (.62 and .69, respectively) while their caution indices are moderate and low(.43 and .16), so that these students are not flagged for further attention. Yet, these students (like many others in the January dataset) had considerable trouble with addition problems, as evident from Table 4. Therefore, their not being cautioned was inappropriate, and was due only to their errors' being fairly common for their group.

To confirm the above interpretations, let us now look at the NCI and C_1 values for these three students when the items are arranged in difficulty order for the November dataset as modified in the manner described earlier. That is, we now inquire how anomalous each of these students' response patterns would look if they had been members of the November group, who were close to mastery stage. The answer is, the NCI for Student #30 becomes the largest of three (.67), while Students #37

Table 3

Response Patterns of Four Subtests for Students #37, #12 and #30

January Order		S#37	S#12	S#30	Nov. Modified		S#37	S#12	S#30
-16 - (-7)	8	0000	1110	1111	12 + -3	3	1111	1010	1111
2 - 11	16	0100	1110	0000	-14 + -5	10	1111	0010	0000
12 + -3	3	1111	1010	1111	-3 + 12	5	1011	1111	1111
8 - 6	7	0100	1111	1111	3 + -5	11	1111	1111	0111
-6 + 4	15	1111	1101	1111	-6 + 4	15	1111	1101	1111
-3 + 12	5	1011	1111	1111	8 - 6	7	0100	1111	1111
3 + -5	11	1111	1111	0111	-16 - (-7)	8	0000	1110	1111
-14 + -5	10	1111	0010	0000	-6 - (-8)	1	0000	1010	1000
-6 - (-8)	1	0000	1010	1000	-3 - +12	13	0111	0100	0000
-7 - 9	2	1111	0000	0000	2 - 11	16	0100	1110	0000
-3 - +12	13	0111	0100	0000	1 - (-10)	4	0000	0000	0000
1 - (-10)	4	0000	0000	0000	-7 - 9	2	1111	0000	0000
Total Score		28	27	24			28	27	24
Caution Index		.93	.43	.16			.30	.60	.44
NCI		-.10	.62	.69			.49	.44	.67
ICI		.96	.88	.99			.96	.88	.99

Table 4

Error Patterns for Subject #37, #12 and #30

Subject	Addition				Subtraction			
	Subt 1	Subt 2	Subt 3	Subt 4	Subt 1	Subt 2	Subt 3	Subt 4
S# 37	(11,21)	(11,0)	(11,21)	(11,21)	treat parentheses as absolute value			
S# 12	(13,21)	(13,0)	0	(0,21)	(13,0)	(0,24)	(13,0)	0
S# 30	(13,0)	(13,21)	(13,21)	(13,21)	(13,0)	*(13,21)	*(13,21)	*(13,21)

* = (32)

and 12 have similar values (.49 and .44). Their caution indices are .30, .60 and .44 for Students #37, 12 and 30, respectively. As expected #37 has the smallest caution index, reflecting the fact, already mentioned, that her errors were more typical for students close to the mastery stage than the beginners in the January dataset.

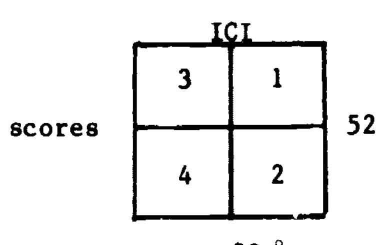
The foregoing illustrative examples should have underscored the undesirability (for error-diagnostic purposes) of the dependence of such indices as NCI and C_i on the particular difficulty order of items that is used in computing them. Clearly, what is needed is an index that does not depend on the item difficulty order for a group but on that for each individual in his/her own right.

The Individual Consistency Index (ICI)

The Individual Consistency Index (ICI) depends on the task difficulties as determined by an individual student's state of knowledge. Its definition calls for the existence of two or more parallel subtests, and the sets of parallel items are arranged in ascending order of task difficulties for the student in question -- i.e., in the order of the student's subscores on the separate tasks such as the 12 shown in Table 3, where each task is represented by four parallel items. The actual calculation of ICI is the same as that for NCI -- or, more precisely, for a group of m subjects, where m is the number of parallel subtests.

Its value is large to the extent that the individual remains in the same state, and hence responds to similar items in the same way -- i.e., using the same rule. If ICI is low, on the other hand, then the individual is probably not sure how the problems can be solved so he/she tries various strategies to respond to the questions. Or, he/she is careless and makes a considerable number of random errors. Thus, ICI can serve better than NCI and the Caution Index as an index for flagging individuals who probably require further attention for remediating their errors and/or making finer diagnoses.

Table 5 is a summary of the error patterns committed by the 127 students in the November data, divided into four subgroups by the following criteria: (1) students whose scores are higher than 52 (the highest total score in Table 1 is 13. Therefore 4 x 13 is taken as a criterion) and ICI values are greater than or equal to .90; (2) those whose scores are lower than or equal to 52 and ICI values are greater than or equal to .90; (3) those who have scores higher than or equal to 52 and have ICI



values lower than .90; (4) those whose scores are lower than or equal to 52 and ICI values are lower than .90.

Insert Table 5 about here

The error types and their frequencies are shown in four sections of the table. As stated in the table, a number sign (#) in front of an error pattern represents the scores adjusted for both addition and subtraction problems (Birenbaum & Tatsuoka, 1980). The question marks in front of the error patterns stand for the scores adjusted for subtraction problems, even though rules were equivocal. The detailed description of adjusting criterion is given in Birenbaum & Tatsuoka. The point is that the modified dataset improved the approach to unidimensionality drastically. Moreover, parameter estimation for the two parameter logistic model by maximum likelihood (using the computer program GETAB written by Robert Baillie) was processed successfully with the modified dataset while the original dataset failed to yield finite estimates.

It is interesting to note that Category 2 contains most erroneous patterns with high consistency. On the other hand, Categories 1 and 3 consist of patterns in which the right rule appears at least once and the other elements are zeros. The zeros mean that either the response patterns are inconsistent or that rules are so complicated that our error-diagnostic system could not determine them specifically. But from the error patterns, and a close examination of the generated error vectors in our error-diagnostic system, the zeros in Categories 1 and 2 are mostly due to random errors. Therefore we can safely conclude that the structure of the unidimensional subset consists of the responses that result from using the right rule, plus random errors.

Replication of the Result Obtained from the November Data

The November data were obtained from 8th graders in 1979. A year later, a new group of 161 8th graders took a 64-item signed-number addition and subtraction test that was parallel to the November test. The test was administered right after the teachers (who also taught the students in the November data) completed their instruction. The teachers used exactly the same teaching method, materials and quizzes as those of the last year. The only difference in the new data, the March data, is that the numbers in the 16 tasks are slightly changed as shown in Appendix V. Table 6 is a summary of the classification of error patterns by ICI and total scores.

Insert Table 6 about here

The result again indicates that ICI is a useful index for extracting a unidimensional dataset by classifying the erroneous patterns into Category 2. Also the data structure of the extracted unidimensional subset in Table 6 confirms that the responses consist of

Table 5

Error Patterns Causing a Mess in Dimensionality
of Nine Subtraction Tasks (36 items)

Category 1

Error Pattern	Frequency
(1)(1)(1)(1)	15
0 (1)(1)(1)	5
(1) 0 (1)(1)	3
(1)(1) 0 (1)	2
(1)(1)(1) 0	2
0 0 (1)(1)	2
0 (1)(1) 0	1
(1) 0 (1) 0	1
0 (1) 0 (1)	1
(1) 0 0 0	2
0 (1) 0 0	1
0 0 (1) 0	3
(1) 0 0 (1)	1
(40)(1)(1)(1)	1
0 0 0 0	1
Total No. of Stu.	41

Category 2

	Error Pattern	Frequency
	(5)(5)(5)(5)	4
	0 (5)(5)(5)	1
#	0 0 (5)(5)	1
#	(16)(16)(40)(16)	1
	(5)(40)(5)(5)	1
#	(16)(16)(5)(16)	1
#	(16)(16)(16) 0	1
#	(16)(40)(16)(16)	1
#	(40)(40)(40)(40)	2
	(40) 0 (5)(5)	1
#	0 (40)(16)(5)	1
#	(5)(40)(40)(40)	1
#	(40)(40)(40)(16)	2
#	0 (40)(16)(16)	1
#	(40)(40)(23)(23)	1
#	(19)(19)(19)(19)	1
#	0 (19)(19)(19)	1
#	0 0 (40)(40)	1
	0 (40) 0 0	1
#	(6)(6)(6)(6)	1
#	(15)(15)(15)(15)	2
#	(4)(4)(4)(4)	1
#	(7)(7)(7)(7)	1
#	(43)(43)(43)(43)	2
#	0 0 0 (5)	1
	Total No. of Stu.	32

#Scores adjusted

*Performance on addition problems was not consistent

?Scores adjusted for subtraction problems, even tnough rule were equivocal

Category 3

Error Pattern	Frequency
(1)(1)(1)(1)	5[a]
0 (1)(1)(1)	5
(1) 0 (1)(1)	4
(1)(1) 0 (1)	2
(1)(1)(1) 0	1
0 0 (1)(1)	3
0 (1) 0 (1)	3
(1)(1) 0 0	2
0 (1)(1) 0	1
(1) 0 (1) 0	1
0 (1) 0 0	1
0 0 0 (1)	4
0 0 (1) 0	1
(40) 0 0 0	1
0 0 0 0	1
Total No. of Stu.	5

Category 4

	Error Pattern	Frequency
	0 0 0 0	7
?	0 0 0 0	1
?	0 0 0 (1)	1
?	0 0 (1) 0	1
	0 (1) 0 0	1
?	0 0 0 (5)	1
	0 0 (23)(23)	1
#	0 0 (40)(40)	1
#	0 0 0 (19)	1
	0 0 (11)(1)	1
	0 (1) 0 (1)	1*
#	0 (1)(1)(1)	1*
	0 (1)(1)(1)	1
	Total No. of Stu.	15

(1) means that the right rule is used for answering to subtraction problems.

[a] Since ICI is calculated over 16 tasks, if performance on addition problems is not consistent, then the error pattern falls into Category 3 even though the error pattern for subtraction is identical.

Table 6

Replication of the result obtained from the November Data: N = 161, Classification of Bugs

Category 1

Error Pattern	Frequency
(1)(1)(1)(1)	32
0 (1)(1)(1)	7
(1) 0 (1)(1)	5
(1)(1) 0 (1)	6
(1)(1)(1) 0	4
(1)(1) 0 0	1
(1) 0 (1) 0	1
(1) 0 0 (1)	1
0 (1)(1) 0	3
0 0 (1)(1)	2
0 (1) 0 (1)	1
(1) 0 0 0	1
0 0 0 (1)	1
0 0 0 0	2
(1)(11)(1)(11)	1
Total No. of Stu.	68

Category 3

Error Pattern	Frequency
(1)(1)(1)(1)	2
(1)(1) 0 (1)	1
0 (1)(1)(1)	2
(1)(1) 0 0	2
0 (1)(1) 0	7
0 0 (1)(1)	2
(1) 0 (1) 0	1
(1) 0 0 (1)	1
(1) 0 0 0	3
0 (1) 0 0	1
0 0 (1) 0	4
0 0 0 (1)	5
0 0 0 0	6
Total No. of Stu.	47

Category 2

Error Pattern	Frequency
(5)(5)(5) 0	2
0 (5)(5)(5)	1
0 0 (5)(5)	1
0 0 (40) 0	1
(40) 0 (40) 0	1
0 (40)(40) 0	2
0 0 0 (40)	2
(40)(16)(5)(5)	1
(40)(16)(40)(5)	1
0 (5)(16)(16)	1
0 (16)(16) 0	1
0 0 0 0	6
(19)(19)(19)(19)	1
0 0 (19)(19)	1
0 (19)(19) 0	2
0 (3)(3)(3)	1
(11)(11)(11)(11)	1
0 (43)(43)(43)	1
(4)(4)(4)(4)	1
(4) 0 0 (4)	1
Total No. of Stu.	29

Category 4

Error Pattern	Frequency
0 (1) 0 (1)	1
0 0 0 (1)	2
0 0 0 (40)	1
0 0 (40) 0	2
0 (5) 0 (5)	1
0 0 0 0	10
Total No. of Stu.	17

answers gotten by using the right rule and of random errors, which is the identical result obtained from the November data.

Insert Table 7 about here

Table 7 shows the eigenvalues of the March data before and after ICI operation was applied. The unidimensionality of the subset improved but not as much as that of the November data. Since the proportion of the number of subjects in Category 2 is 1/4 in the November data while it is 1/8 in the March data, the difference in the increments of the variance accounted for by the first eigenvalues in the two datasets may be explained. The March data, both the original and those modified by ICI yielded convergence when GETAB was used to estimate the parameters of the IRT model.

Conclusion

One of the important areas in the theory and practice of educational measurement is that of criterion-referenced testing. The measurement theories that have been utilized in measuring and evaluating the outcomes of treatments (or instruction) typically depend on binary scores obtained from test items. It was customarily assumed that the underlying structure of the dataset from a criterion-referenced test consists of one major common factor because the items are usually selected from a single content domain. However, several studies have shown that the assumption of unidimensionality should be closely examined even for criterion-referenced tests. This study demonstrated that the dimensionality of a dataset will be affected by the examinee's underlying cognitive processes as well as by the nature of the content domain. The fact is that, after the response patterns yielded by erroneous rules of operation are deleted from the original dataset, the remaining subset of data becomes more nearly unidimensional, and this subset of course consists of responses yielded by the right rule and non-systematic errors. This observation points to an answer to the question of when and why the dataset of an achievement test satisfies the condition of unidimensionality.

Tables 5 and 6 show that ICI is an effective index for detecting erroneous rules of oeration. However, it requires repeated measures. Since most tests don't have several parallel items to measure the performance of a single task, applicability of ICI to these tests will be limited. The drawback of this limitation mus be removed, and a solution to this problem is being developed.

Table 7

Eigenvalues and their variances of the replicated datasets

	The Original Dataset N = 161		Subset in which low score-high ICI subjects are deleted. N = 137	
Factor	Eigenvalues	Variance	Eigenvalue	Variance
1	9.2928	58.0802	9.4820	59.2627
2	1.7834	11.1462	1.3967	8.7295
3	1.0883	6.8018	0.9202	5.7511
4	0.8437	5.2729	0.8070	5.0439
5	0.6985	4.3659	0.5943	3.7141
6	0.4301	2.6879	0.5155	3.2217
7	0.3728	2.3303	0.4466	2.7911
8	0.3331	2.0819	0.3959	2.4742
9	0.2428	1.5178	0.2905	1.8155
10	0.1857	1.1606	0.2807	1.7543
11	0.1747	1.0918	0.2344	1.4649
12	0.1458	0.9115	0.1806	1.1286
13	0.1312	0.8197	0.1585	0.9906
14	0.1172	0.7323	0.1171	0.7318
15	0.0868	0.5423	0.0999	0.6243
16	0.0731	0.4569	0.0803	0.5018

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Appendices

I. The signed-number test used in the January experiment

II. A list of the bugs found in the signed-number addition and subtraction problems

III. Observed complete erroneous rules of operations, their descriptions and codes given in Appendix II

IV. Forty-five rules described in Appendix IIIand their component scores based on the test in Appendix V

V. The signed-number test used in the March experiment

Appendix I
The Signed-Number Test used in the January Experiment

Test Items			
I	II	III	IV
1. -6-(-8)=2	17. -1-(-10)=9	33. -3-(-5)=2	49. -2-(-11)=9
2. -7-9=-16	18. -2-11=-13	34. -4-6=-10	50. -5-14=-19
3. 12+-3=9	19. 7+-5=2	35. 15+-6=9	51. 4+-2=2
4. 1-(-10)=11	20. 3-(-12)=15	36. 5-(-7)=12	52. 6-(-8)=14
5. -3+12=9	21. -1+10=9	37. -4+13=9	53. -2+11=9
7. 8-6=2	23. 7-5=2	39. 4-2=2	55. 9-7=2
8. -16-(-7)=-9	24. -12-(-10)=-2	40. -11-(-2)=-9	56. -7-(-5)=-2
10. -14+-5=-19	26. -10+-1=-11	42. -7+-5=-12	58. -10+-8=-18
11. 3+-5=-2	27. 2+-11=-9	43. 6+-8=-2	59. 1+-10=-9
12. 13-(-4)=17	28. 0-(-9)=9	44. 6-(-4)=10	60. 0-(-2)=2
13. -3-+12=-15	29. -2-+11=-13	45. -7-+9=-16	61. -4-+6=-10
15. -6+4=-2	31. -5+3=-2	47. -4+2=-2	63. -8+6=-2
16. 2-11=-9	32. 5-14=-9	48. 7-16=-9	64. 4-13=-9

Appendix II

A List of "Bugs" Found in the Sign-numbered Addition and Subtraction Problems (Verbal Rules)

	Code	Incomplete Rule (in terms of sign operation)
1.	21	Taking the sign of the larger absolute number to answers (i.e., take the correct sign in addition problems.)
2.	22	Taking the sign of the smaller absolute number to answers
3.	23	Taking always a + sign to answers
4.	24	Taking always a - sign to answers
5.	25	Taking the sign of the first number
6.	26	Taking the sign of the second number
7.	27	Taking the sign of the product of the two numbers
8.	28	Taking the sign of the larger number (as an integer) to answers
9.	29	Taking the sign of the smaller number (as an integer) to answers.
10.	210	Treat the operation sign as the sign of the second number, <u>if</u> the sign of the second number is explicit. Then take the sign of the larger absolute values to answers.
11.	211	Besides the bug described in 210, if the sign of the larger absolute value is the first number with an implicit sign, then take the explicit sign or the sign obtained by the 210 bug to answers
12.	212	The same as bug 210 except take the sign of the larger integer.
13.	213	Basically follow the right rule, but when the sign of the larger absolute value is implicit and also is the second number, then take the sign of the other number to answers.

(cont.)

Appendix II (cont.)

	Code	Incomplete Rule (in terms of sign operation)
14.	214	Basically follow the bug of code 26, but when the larger integer is the second number and its sign is implicit, then take the sign of the other number.
15.	215	For the addition problems, if the sign of the second number is implicit, then apply the regular arithmetic operation. For example, -L+S has a wrong sign. For the subtraction problems, the implicit sign of the second number remains unchanged when they converted to addition problems, and regular arithmetic operation is applied but -L+S type has a wrong sign.
16.	216	Change the explicit sign of the second number, then take the sign of the larger absolute value.
17.	217	Change the explicit sign of the second number, then take the sign of the larger integer
18.	218	Change the sign of the second number and take the sign of the larger absolute value. Treat all problems as subtraction problems.
19.	219	Change the sign of the second number and take the sign of the larger integer value. Treat all problems as subtraction problems.

Appendix II (cont.)

	Code	Incomplete Rules (in terms of absolute value operation)
1.	11	If the signs of two numbers are same, then add the absolute values of the two numbers. If the signs of two numbers are different, then subtract the smaller absolute value from the larger absolute value
2.	12	In spite of different skill types, the absolute values of the two numbers are always added.
3.	13	No matter what the skill types are, the smaller absolute value is subtracted from the larger absolute value.
4.	14	Opposite of the right operation, i.e., if the signs of two number are same, then subtract, $\|L\| - \|S\|$. If the signs of two numbers are different then add the absolute values of the two numbers, $\|L\| + \|S\|$.
5.	15	If the sign of the first number is positive, +, then $\|L\| + \|S\|$. If the sign of the first number is negative, -, then $\|L\| - \|S\|$.
6.	16	If the sign of the second number is positive, +, then $\|L\| + \|S\|$ and if the sign of the second number is negative, -, then $\|L\| - \|S\|$.
	17	If the sign of the first number is positive, then $\|L\| - \|S\|$ and if it is negative, then $\|L\| + \|S\|$.
8.	18	If the absolute value of the first number is larger, then $\|L\| + \|S\|$ and if it is smaller, then $\|L\| - \|S\|$.
9.	19	If the absolute value of the first number is larger, then $\|L\| + \|S\|$.
10.	110	If the first number is larger as an integer, then $\|L\| + \|S\|$. If it is smaller then $\|L\| - \|S\|$.
11.	111	If the first number is larger as an integer, then $\|L\| - \|S\|$ and if its not then add the two absolute values.
12.	112	Changing the sign of the second number, then applying the right rule 11.

Appendix II (cont.)

Subtraction Problems

(Problems in the conversion of subtraction to addition)

	Code	
1.	31	Convert the operation sign - to + and change the sign of the second number.
2.	32	Convert the operation sign - to + and don't change the sign of the second number
3.	33	Convert the operation sign - to + and change the sign of the first number
4.	34	Convert the operation sign - to + and change both the signs of the two numbers
5.	35	Convert the operation sign - to + and change the sign of the second number. At the same time, if the sign of the first number is negative, -, then it will be changed to +.
6.	36	Convert the operation sign - to + but don't change the sign of the second number. At the same time, if the sign of the first number and second numbers are negative, then change the signs to +. Thus, task -L - (-S) is converted to +L + (+S).

Appendix II (cont.)

Skill Types		Conv. 31	Original 31		Conv. [illegible]		Conv. 33		Conv. 34		Conv. 35	
12+-3	3	12+-3	L+-S									
-3+12	5	-3+12	-S+L									
-14+-5	10	-14+-5	-L+-S									
3+-5	11	3+-5	S+-L									
-5+-7	14	-5+-7	-S+-L									
-6+4	15	-6+4	-L+S									
-6-(-8)	1	-6+(+8)	-S-(-L) → -S+(+L)	5	-S+(-L)	14	+S+(-L)	11	S+(+L)	0	+S+(+L)	0
-7-9	2	-7+-9	-S-L → -S+-L	14	-S+L	5	+S+L	0	S+-L	11	S+-L	11
1-(-10)	4	1+(+10)	S-(-L) → S+(+L)	0	S+(-L)	11	-S+(-L)	14	-S+(+L)	5	S+(+L)	0
8-6	7	8+-6	L-S → L+-S	3	L+S	0	-L+S	15	-L+-S	10	L+-S	3
-16-(-7)	8	-16+(+7)	-S-(-S) → -L+(+S)	15	- +(-S)	10	L+(-S)	3	+L+(+S)	0	L+(+S)	0
-12-3	9	-12+-3	-L-S → -L+-S	10	-L+S	15	L+S	0	+L+-S	3	L+-S	0
9-(-7)	12	9+(+7)	L-(-S) → L+(+S)	0	L+(-S)	3	-L+(-S)	10	-L+(+S)	15	L+(+S)	0
-3-+12	13	-3+-12	-S-+L → -S+-L	14	-S++L	5	S++L	11	+S+-L	11	S+L	11
2-11	16	2+-11	S-L → S+-L	11	S+L	0	-S++L	14	-S+-L	14	S+-L	11
-13-+4		-13+-4	-L-+S → -L+-S	10	-L++S	10	L++S	3	+L+-S	3	L+-S	3
13-+15		13+-15	S-+L → S+-L	11	S++L	0	-S++L	5	-S+-L	14	S+-L	11

0: L+S = S+L

Appendix II (cont.)

Subtraction Problems
Having Hidden Signs and Parentheses
(Interpretation of the problems)

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hs	When the sign of the second number is implicit, the sign of the second number is ignored. Thus, the sign of the second number remains the same although the operation sign "-" is changed to "+". This error appears in all 4 types of problems, L - S, S - L, -L - S and -S - L.
ah	In the above mentioned case, L - S and S - L are carried out by regular arithmetic but -L + S and -S - L will be converted to -L + S and -S + L respectively.
a	L - S and S - L are treated as regular arithmetic problems. The implicit signs of the skill types -L - S, -S - L are not ignored. So they are changed to -L + -S -S + -L respectively.
Pbf	The parentheses of the problems are interpreted as absolute value symbols before any conversion of subtraction to addition.
Paf	The parentheses are treated as absolute value symbols after any conversion occurs.
ai	Besides L - S and S - L types, students apply regular arithmetic operations to (-L) - (-S) and (-S) - (-L) types and get wrong signs for S - L and/or (-S) - (-L).
ho	The operation sign in the task -L - S is recognized as the sign of the second number. So when a student converts subtraction problems to addition problems, insert a + sign before the second minus sign, right after the larger number L. So this task will yield the task type, -L + -S.

Appendix III

Observed Complete Erroneous Rules of Operations

Their Descriptions and Codes Given in Appendix II

1. (11,21)(31)(11,21)

The right rule for addition problems, (11,21) -- add the absolute values of two numbers if the signs of the numbers are same, or subtract the smaller absolute value from the larger one if the signs are alike and take the sign of the larger number to the answer. Then convert subtraction problems into addition ones by changing the operation sign -to + and the sign of the second number to a opposite sign. Carry out the right addition operation on the newly converted additon problems.

2. (11,21)(31)Paf(11,21)

The right rules for addition problems, and right conversion (31) is carried out but the numbers in the parenthesis are changed to positive numbers, then the right rule for addition is used for the new addition problems.

3. (11,21)Pbf(31)(11,21)

The numbers in the parenthesis are changed to positive before subtraction problems are converted to addition problems.

4. (11,25)(31)(11,25)

The student takes the right absolute value in answers for addition problems. For subtraction problems, the student converts subtraction to addition correctly but applies the same wrong rule.

5. (11,21)(32)(11,21)

The right rule, for taking the proper absolute value and the sign of the larger absolute value, is used for addition problems. For subtraction problems a student changes the operation sign of - to + without changing the sign of the second number then applies the right rule for addition problems. The code is expressed by (11,21) for addition, conversion error (32) and the right rule (11,21) again. Thus, this bug is (11,21)(32)(11,21).

6. (11,21)(32)a(11,21)

The addition problems are right, but subtraction problems are converted in a wrong way -- by changing operation signs, -, to plus, +, except for the problems types L - S and S - L. The latter two tasks are answered by a regular arithmetic method. The other converted tasks are answered by using the right rule.

7. (11,21)(32)ha(11,21)

Apply the right addition rule and converts most subtraction problems by applying (32) -- changes operation signs to plus but doesn't change the signs of the second number -- but -L - S type. Operation + is inserted right after the larger number L. So the problem becomes -L + -S type.

8. (11,21)(32)hoa(11,21)
This bug is a combination of bugs (11,21)(32)a(11,21) and (11,21)(32)ho(11,21). That is, -L - S type was changed to -L + -S, and, moreover, L - S and S - L types were answered by a regular arithmetic method.

9. (11,21)(36)(11,21)
After all subtraction problems are converted to addition problems according to the rule (36)--convert the operation sign - to +, without changing the sign of the second number. At the same time, if there are three minuses such as -L-(-S), then all minuses will be plus. Thus +L+(+S) is the converted problem type.

10. (11,21)(33)(11,21)
Convert subtraction problems into addition problems but change the sign of the first number instead of the second number. The right addition rule was applied before and after the conversion.

11. (11,21)(31)hs(11,21)
When subtraction problems are converted to addition, hidden signs of the second numbers are ignored, so they are not changed to negative. Thus -L - S, -S - L, S - L, L - S resulted in wrong answers.

12. (11,21)(31)hsa(11,21)
L - S and S - L types are answered by a regular arithmetic method without being converted, but - L - S and -S - L are converted to - L + S and -S + L, resulting in wrong answers.

13. (11,21)(32)Pbf(11,21)
Before subtraction problems are converted, the parenthesis in the problems are considered as an absolute value notation and the numbers in the parenthesis are changed to positive numbers before the conversion (32) is taken .

14. (11,21)(32)Paf(11,21)
The numbers in the parenthesis are changed to be positive numbers after subtraction problems are converted to addition problems according to the wrong rule (32).

15. (11,21)(32)hoPaf(11,21)
-L - S and -S - L types resulted in right conversion even though the wrong conversion rule (32) was applied. Then the problems with parenthesis are changed to be positive.

16. (11,21)(13,21) {or (11,21)(32)(13,21)}
A student used the right rule for addition problems but he/she subtracted two numbers, |L| - |S|, and took the sign of the larger absolute value.

17. (11,21)Paf(13,21)
After carrying out the rule 13, subtracting two numbers, the numbers in the parenthesis are changed to positive numbers then the rule (21) is used for taking the sign to answers.

18. (11,21)ho(13,21)
For -L - S type, having a hidden sign for the second number, a student treated operation sign - as the sign of the second number.

19. (12,21)(31)(12,21)
Subtraction problems are converted to addition problems by the right conversion rule but an erroneous rule for addition problem, (12,21) are applied consistently both before and after the conversion.

20. (13,21)(31)(13,21)
The operation of converting subtractin to addition is carried out correctly but a wrong rule (13,21)-- always subtracting two numbers, |L| - |S| and taking the sign of the larger absolute value-- was applied throughout the problems.

21. (11,?1)(12,24)
The conversion of subtraction was not carried out. For addition, the right rule was used, but for subtraction problems, two numbers are added and a minus sign was taken to the answers.

22. (11,21)(13,24)
The conversion of subtraction was not carried out. The rule (11,21) for addition, the rule (13,24) for subtraction problems.

23. (13,21)(13,21)
The conversion of subtraction was omitted and two numbers are always subtracted and the sign of the larger absolute value was always taken to answers.

24. (13,21)(13,24)
The conversion was omitted and erroneous rules (13,21) for addition problems, (13,24) for subtraction problems are used.

25. (13,21)(32)a(11,21)
For addition problems, the wrong rule (13,21) is used. The conversion is again wrong (32). L - S and S - L are answered by a regular arithmetic method. The rest of the newly converted addition problems are answered by the right rule

26. (13,24)(13,24)
The same rule, |L| - |S| and the sign, -, to answers.

27. (12,24)(13,24)
The conversion operation is ignored. Add two numbers for addition, subtract two numbers |L| - |S| for subtraction problems. The sign, -, is always taken to answers.

28. (12,23)(13,24)
No conversion is made. The two numbers are added for addition problems and subtracted for subtraction problems. The sign, +, is always taken to answers for addition while the sign, -, is always taken for subtraction problems.

29. (13,23)(13,24)
Nc conversion is made. The two numbers are always subtracted, |L| - |S| but + for addition problems and, -, for problems are taken to answers.

30. (13,25)(13,25)
No conversion is made. The two numbers are always subtracted, |L| - |S| and the sign of the first number is taken to answers.

31. (13,25)(13,24)
No conversion is made. The two numbers are always subtracted |L| - |S|. The sign of the first number is taken to answers for addition problems and a minus sign is always taken for subtraction problems.

32. (13,23)(13,23)
No conversion is made. The two numbers are always subtracted, |L| - |S|. The sign, +, is always taken to answers.

33. (12,25)(13,25)
No conversion is made. The two numbers are added for addition and subtracted for subtraction problems. The sign of the first numbber is taken to answers for both the addition and subtraction problems.

34. (15,25)(15,25)
The operation of converting subtraction to addition problems is ignored. Instead, this student added two numbers, |L| + |S|, if the sign of the first number is positive. He subtracted, |L| - |S| if the sign of the first number is negative. He used this rule for both the addition and subtraction problems.

35. (15,25)(13,24)
For addition problems, the same rule as the previous example is applied but for subtraction problems, he subtracts two numbers, |L| - |S| and takes a minus sign.

36. (16,23)(13,23)
For addition problems, if the sign of the second number is positive then two absolute values are added, |L| + |S| and if it is negative, then the two absolute values are subtracted, |L| - |S|. For subtraction problems, the two numbers are always subtracted and a positive sign, +, is always chosen for the sign to answers.

37. (16,26)ho(16,26)
For addition problems, the absolute values of answers are the same as the previous example of (16,23). But the sign of the answers is the sign of the second number. For subtraction problems, the same rule as addition is used, but -L - S or -S - L types where the sign of the second number is implicit, then operation sign is considered as the sign of the second number.

38. (17,21)(17.21)
For addition and subtraction problems, if the sign of the first number is +, then |L| - |S| and if it is negative, then |L| + |S|. But the sign of the larger absolute value is taken to answers.

39. (12,21)(13,21)
For addition problems, absolute values of the two numbers are always added and the sign of the larger number is taken to answers. For subtraction prob s, two absolute values are always subtracted and the sign of the large number is taken to answers.

40. (12,21)Ai (13,21)
The right rule for addition problems. For subtraction problems, L - S, S - L, -L - (-S) and -S - (-L) types are answered by using a regular arithmetic method but the signs of answers are not always correct. All other problems are answered by (13,21)-rule. No conversion of subtraction to addition problems was made.

41. (32)(11,21)(31)(11,21)
Subtraction problems are correctly converted by the rule (31), and answered correctly by using the right rule of addition problems. But addition problems are also converted by changing the sign of the second number and answered by the right rule.

42. (13,21)(31)(12,21)
In addition problems, the student finds the difference between the two numbers and assigns the sign of the larger absolute value to the result. In subtraction, converts subtraction to addition by Rule (31) which is right, then adds the two absolute values and takes the sign of the larger absolute value from the converted addition problems to answers.

43. (11,27)(13,24)
If two numbers have two similar signs, then the two absolute values are added and sign of multiplication is taken to the result. If two numbers have different signs, then |L| - |S| as the absolute value and a minus sign to the answer.

44. (11,21)(34)(11,21)
In subtraction, the student changes the signs of both numbers as well as the operation sign and carries out the right rule for addition problems.

45. (11,21)(36)(13,21)
In subtraction, the two absolute values are subtracted and the sign of the larger number is taken to the result after conversion rule (36) is applied.

Appendix IV

45 Rules Described in Appendix III and Their Components Scores Based on the Test in Appendix V

Item #	Task	3	4	5	6	7/8	9	10	11	12	13	14	15	16
3	14+–7=+7													
5	-4+13=+9													
10	-16+–3=-19													
11	2+–8=-6													
14	-6+–9=-15													
15	-8+5=-3													
		a b c												
1	-3-(-7)=+4	0 0 0	1 1 1	0 0 0	0 0 0	0 0 0	1 0 0	0 1 0	1 1 1	1 1 1	0 0 0	1 1 1	1 1 1	0 1 0
2	-2-8=-10	1 1 1	0 0 0	0 0 0	0 0 0	1 1 1	0 0 0	0 1 0	0 0 0	0 0 0	0 0 0	0 0 0	1 1 1	0 0 0
4	5-(-12)=+17	0 0 0	1 1 1	0 0 0	0 0 0	0 0 0	0 0 0	0 1 0	1 1 1	1 1 1	0 0 0	1 1 1	1 1 1	0 0 0
6	-11-+8=-19	1 1 1	1 1 1	1 0 0	1 0 0	1 0 0	1 0 0	0 1 0	1 1 1	1 1 1	1 0 0	1 0 0	1 0 0	1 0 0
7	9-4=+5	1 1 1	1 0 0	1 0 0	1 1 1	1 1 1	1 0 0	0 1 0	1 0 0	1 1 1	1 0 0	1 0 0	1 1 1	1 1 1
8	-15-(-9)=-6	1 0 0	1 1 1	1 0 0	1 0 0	1 0 0	0 0 0	0 1 0	1 1 1	1 1 1	0 1 0	1 1 1	1 1 1	1 1 1
9	-13-5=-18	1 1 1	1 0 0	1 0 0	1 0 0	1 1 1	1 0 0	0 1 0	1 0 0	0 1 0	1 0 0	0 0 0	1 1 1	1 0 0
12	8-(-6)=+14	1 0 0	1 1 1	1 0 0	1 0 0	1 0 0	1 0 0	0 1 0	1 1 1	1 1 1	1 0 0	1 1 1	1 1 1	1 0 0
13	-5-+11=-16	1 1 1	1 1 1	0 0 0	0 0 0	0 0 0	0 0 0	0 1 0	1 1 1	1 1 1	0 0 0	0 0 0	0 0 0	0 0 0
16	1-10=-9	1 1 1	0 0 0	0 0 0	1 1 1	1 1 1	0 0 0	0 1 0	0 0 0	1 1 1	0 0 0	0 0 0	1 1 1	0 1 0

a Sign component scores are in the first column
b Absolute value component scores are in the second column
c Regular scores (multiplication of the first and second numbers)
* Response patterns of Rule 40 are obtained by assuming S-L type has a wrong sign.

Appendix IV (cont.)

17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
		0 0 0	0 1 0	1 1 1	1 1 1	0 1 0	0 1 0	0 1 0	1 1 1	1 0 0	0 0 0	0 1 0	0 1 0	0 1 0
		1 0 0	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	0 1 0	0 0 0	1 0 0	1 1 1	0 1 0	0 1 0
		1 1 1	1 0 0	1 1 1	1 1 1	1 0 0	1 0 0	1 0 0	1 0 0	1 1 1	0 1 0	0 0 0	1 0 0	1 0 0
		1 0 0	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 0 0	0 0 0	0 1 0	0 1 0	0 1 0
		1 1 1	1 0 0	1 1 1	1 1 1	1 0 0	1 0 0	1 0 0	1 0 0	1 1 1	0 1 0	0 0 0	1 0 0	1 0 0
		1 0 0	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 0 0	0 0 0	0 1 0	1 1 1	1 1 1
1 1 1	0 1 0	1 0 0	1 1 1	0 0 0	0 1 0	0 1 0	0 1 0	0 0 0	0 1 0	0 1 0	0 1 0	0 1 0	0 1 0	0 1 0
0 0 0	1 0 0	1 1 1	1 0 0	1 1 1	1 0 0	0 0 0	1 0 0	0 0 0	1 0 0	1 0 0	1 0 0	1 0 0	1 0 0	1 0 0
1 0 0	0 0 0	1 1 1	1 0 0	0 1 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	1 0 0	0 0 0
1 0 0	1 0 0	1 1 1	1 0 0	1 1 1	1 0 0	1 0 0	1 0 0	1 0 0	1 0 0	1 0 0	1 0 0	1 0 0	1 0 0	1 0 0
1 1 1	1 1 1	1 0 0	1 1 1	0 0 0	0 1 0	1 1 1	0 1 0	1 1 1	0 1 0	0 1 0	0 1 0	0 1 0	1 1 1	0 1 0
1 1 1	1 1 1	1 0 0	1 1 1	1 0 0	1 1 1	1 1 1	1 1 1	1 0 0	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1
1 0 0	1 0 0	1 1 1	1 0 0	1 1 1	1 0 0	1 0 0	1 0 0	1 0 0	1 0 0	1 0 0	1 0 0	1 0 0	1 0 0	1 0 0
1 0 0	1 0 0	1 1 1	1 0 0	0 1 0	0 0 0	1 0 0	0 0 0	1 0 0	0 0 0	0 0 0	0 0 0	0 0 0	1 0 0	0 0 0
0 0 0	0 0 0	1 1 1	1 0 0	1 1 1	1 0 0	0 0 0	1 0 0	0 0 0	1 0 0	1 0 0	1 0 0	1 0 0	1 0 0	1 0 0
0 1 0	0 1 0	1 0 0	1 1 1	1 0 0	1 1 1	0 1 0	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	0 1 0	1 1 1

32	33	34	35	36	37	38	39	40*	41	42	43	44	45
0 1 0	0 0 0	0 0 0	0 0 0	0 1 0	1 1 1	0 1 0	0 0 0	1 1 1	0 0 0	0 1 0	1 1 1	1 1 1	1 1 1
1 1 1	1 0 0	0 1 0	0 1 0	1 0 0	1 0 0	1 0 0	1 0 0	1 1 1	0 0 0	1 1 1	0 1 0	1 1 1	1 1 1
0 0 0	1 1 1	1 0 0	1 0 0	0 0 0	1 0 0	1 1 1	1 1 1	1 1 1	1 0 0	1 0 0	0 1 0	1 1 1	1 1 1
0 1 0	0 0 0	0 0 0	0 0 0	0 1 0	1 1 1	1 1 1	1 0 0	1 1 1	0 0 0	1 1 1	1 1 1	1 1 1	1 1 1
0 0 0	1 1 1	1 0 0	1 0 0	0 0 0	1 0 0	1 1 1	1 1 1	1 1 1	0 0 0	1 0 0	0 1 0	1 1 1	1 1 1
0 1 1	1 0 0	1 1 1	1 1 1	0 0 0	1 0 0	1 0 0	1 0 0	1 1 1	1 0 0	1 1 1	1 1 1	1 1 1	1 1 1
1 1 1	0 1 0	0 1 0	0 1 0	1 1 1	0 1 0	0 0 0	0 1 0	1 1 1	1 1 1	1 0 0	1 0 0	1 0 0	1 0 0
0 0 0	1 0 0	1 0 0	1 0 0	0 0 0	1 0 0	1 1 1	0 0 0	0 0 0	1 1 1	1 1 1	1 0 0	1 0 0	1 1 1
1 0 0	1 0 0	1 1 1	0 0 0	1 0 0	0 0 0	0 0 0	0 0 0	0 0 0	1 1 1	1 1 1	0 0 0	1 0 0	0 0 0
0 0 0	1 0 0	1 0 0	1 0 0	0 0 0	0 1 0	1 1 1	1 0 0	1 0 0	1 1 1	1 1 1	1 0 0	0 1 0	1 0 0
1 1 1	1 1 1	1 0 0	0 1 0	1 1 1	0 1 0	1 1 1	1 1 1	1 1 1	1 1 1	1 0 0	1 0 0	0 0 0	1 0 0
0 1 0	1 1 1	1 1 1	1 1 1	0 1 0	1 1 1	1 0 0	1 1 1	1 1 1	1 1 1	1 0 0	0 0 0	0 0 0	0 0 0
0 0 0	1 0 0	0 0 0	1 0 0	0 0 0	1 0 0	1 1 1	1 0 0	1 0 0	1 1 1	1 1 1	1 0 0	0 0 0	1 1 1
1 0 0	1 0 0	1 1 1	0 0 0	1 0 0	0 0 0	1 0 0	1 0 0	1 0 0	1 1 1	1 1 1	0 0 0	0 0 0	1 0 0
0 0 0	1 0 0	1 0 0	1 0 0	0 0 0	0 1 0	0 1 0	0 0 0	0 0 0	1 1 1	1 1 1	1 0 0	1 0 0	0 0 0
0 1 0	0 1 0	0 0 0	1 1 1	0 1 0	1 1 1	0 1 0	0 1 0	0 1 0	1 1 1	1 0 0	0 0 0	1 0 0	0 0 0

Appendix V

The Signed-Number Test

The March data

I		II		III		IV	
1.	-3-(-7)=	17.	-2-(-11)=	33.	-2-(-8)=	49.	-3-(-12)=
2.	-2-8=	18.	-3-15=	34.	-3-9=	50.	-6-13=
3.	14+-7=	19.	7+-4=	35.	15+-6=	51.	9+-3=
4.	5-(-12)=	20.	6-(-11)=	36.	4-(-9)=	52.	4-(-7)=
5.	-4+13=	21.	-5+8=	37.	-3+11=	53.	-2+10=
6.	-11-+8=	22.	-12-+4=	38.	-11-+7=	54.	-16-+5=
7.	9-4=	23.	9-6=	39.	8-5=	55.	6-4=
8.	-15-(-9)=	24.	-16-(-10)=	40.	-13-(-4)=	56.	-9-(-5)=
9.	-13-5=	25.	-5-3=	41.	-15-3=	57.	-8-4=
10.	-16+-3=	26.	-11+-3=	42.	-9+-6=	58.	-10+-7=
11.	2+-8=	27.	4+-12=	43.	4+-7=	59.	1+-11=
12.	8-(-6)=	28.	6-(-4)=	44.	11-(-1)=	60.	12-(-5)=
13.	-5-+11=	29.	-4-+11=	45.	-6-+8=	61.	-7-+8=
14.	-6+-9=	30.	-7+-9=	46.	-4+-12=	62.	-4+-15=
15.	-8+5=	31.	-7+2=	47.	-11+3=	63.	-9+5=
16.	1-10=	32.	6-12=	48.	5-13=	64.	6-14=

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